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The first result is the same as the average distance of a point in a circle from a point in its circumference. The second result is the same as the average distance between two points on the circumference of a circle.

Also solved by *F. P. MATZ*. Professor Matz in his solution did not go through the details of integration as Professor Zerr has done.

II. Solution by *L. C. WALKER, A. M.*, Professor of Mathematics, Petaluma High School, Petaluma, Cal.

Let P, Q be the random points, O the center of the hemisphere, and A the vertex. Put $OP=r$, $\angle POQ=\theta$, $\angle AOP=\varphi$, M_1 =average length of the straight line, and M_2 =average length of the arc of a circle, which joins the points.

Now while θ is constant, and $<\frac{1}{2}\pi$, and $\phi<\frac{1}{2}\pi-\theta$, for each position of P , Q may be taken anywhere in the circumference of a small circle whose pole is P , and radius $rsin\theta$.

But when $\theta<\frac{1}{2}\pi$, and $\varphi>\frac{1}{2}\pi-\theta$, or when $\theta>\frac{1}{2}\pi$, and $\phi>\theta-\frac{1}{2}\pi$, for each position of P, Q may be taken anywhere in the arc of a small circle whose pole is P , and length $2rsin\theta[\pi-\cos^{-1}(\cot\theta\cot\phi)]$.

Hence, if θ is of given value, and $<\frac{1}{2}\pi$, the number of ways the two points can be taken is

$$\int_0^{\frac{1}{2}\pi-\theta} 2\pi r\sin\theta \cdot 2\pi r\sin\phi \cdot r d\phi + \int_{\frac{1}{2}\pi-\theta}^{\frac{1}{2}\pi} 2\sin\theta[\pi-\cos^{-1}(\cot\theta\cot\phi)] \cdot 2\pi r\sin\phi \cdot r d\phi \\ = 4\pi r^3(\pi-\theta)\sin\theta.$$

If $\theta>\frac{1}{2}\pi$, the number of ways the two points can be taken is

$$\int_{\theta-\frac{1}{2}\pi}^{\frac{1}{2}\pi} 2\sin\theta[\pi-\cos^{-1}(\cot\theta\cot\phi)] \cdot 2\pi r\sin\phi \cdot r d\phi = 4\pi r^3(\pi-\theta)\sin\theta.$$

Hence, since the whole number of ways the two points can be taken is $4\pi^2 r^4$, we have

$$M_1 = \frac{1}{4\pi^2 r^4} \int_0^{\pi} 4\pi r^3(\pi-\theta)\sin\theta \cdot 2r\sin\frac{1}{2}\theta \cdot r d\theta = \frac{32r}{9\pi}, \text{ and}$$

$$M_2 = \frac{1}{4\pi^2 r^4} \int_0^{\pi} 4\pi r^3(\pi-\theta)\sin\theta \cdot r\theta \cdot r d\theta = \frac{4r}{\pi}.$$

MISCELLANEOUS.

101. Proposed by *G. B. M. ZERR, A. M., Ph. D.*, Professor of Chemistry and Physics in The Temple College, Philadelphia, Pa.

A wire is laid along the surface of a right cone semi-vertical angle β so that it cuts the generators everywhere at a constant angle θ . Find the radius of curvature and radius of torsion.

Solution by the PROPOSER.

Let s =length of wire from origin to any point, ρ =radius of curvature, $1/\sigma$ =radius of torsion. From problem 85, Calculus, No. 4, Vol. VI., we get $x=$

$$r\cos\varphi, y=r\sin\varphi, z=s\cos\beta\cos\theta, \varphi=\frac{\tan\theta}{\sin\beta}\log s, r=s\sin\beta\cos\theta, dx=\cos\varphi dr-r\sin\varphi d\varphi,$$

$$dy=\sin\varphi dr+r\cos\varphi d\varphi, dz=\cos\beta\cos\theta ds, d\varphi=\frac{\tan\theta}{\sin\beta}\cdot\frac{ds}{s}, dr=\sin\beta\cos\theta ds.$$

$$\therefore dx/ds=\sin\beta\cos\theta\cos\varphi-\sin\theta\sin\varphi, dy/ds=\sin\beta\cos\theta\sin\varphi+\sin\theta\cos\varphi,$$

$$dz/ds=\cos\beta\cos\theta, d^2x/ds^2=-\frac{\sin\theta}{s}\left(\sin\varphi+\frac{\tan\theta\cos\varphi}{\sin\beta}\right),$$

$$d^2y/ds^2=\frac{\sin\theta}{s}\left(\cos\varphi-\frac{\tan\theta\sin\varphi}{\sin\beta}\right), d^2z/ds^2=d^3z/ds^3=0,$$

$$d^3x/ds^3=-\frac{\sin\theta\tan\theta}{s^2\sin\beta}\left(\cos\varphi-\frac{\tan\theta\sin\varphi}{\sin\beta}\right), d^3y/ds^3=-\frac{\sin\theta\tan\theta}{s^2\sin\beta}\left(\sin\varphi+\frac{\tan\theta\cos\varphi}{\sin\beta}\right)$$

$$1/\rho^2=(d^2x/ds^2)^2+(d^2y/ds^2)^2+(d^2z/ds^2)^2=\frac{\sin^2\theta}{s^2}\left(1+\frac{\tan^2\theta}{\sin^2\beta}\right).$$

$$\begin{aligned} \frac{1}{\rho^2\sigma} &= \begin{vmatrix} \frac{dx}{ds}, & \frac{dy}{ds}, & \frac{dz}{ds} \\ \frac{d^2x}{ds^2}, & \frac{d^2y}{ds^2}, & \frac{d^2z}{ds^2} \\ \frac{d^3x}{ds^3}, & \frac{d^3y}{ds^3}, & \frac{d^3z}{ds^3} \end{vmatrix} = \frac{dz}{ds}\left(\frac{d^2x}{ds^2}\cdot\frac{d^3y}{ds^3}-\frac{d^3x}{ds^3}\cdot\frac{d^2y}{ds^2}\right) \\ &= \frac{\sin^2\theta\tan\theta}{s^3\sin\beta}\left(1+\frac{\tan^2\theta}{\sin^2\beta}\right)\cos\beta\cos\theta. \end{aligned}$$

$$\therefore 1/\sigma=\frac{\tan\theta\cos\beta\cos\theta}{s\sin\beta}=\frac{\sin\theta\cos\beta}{s\sin\beta}$$

Also solved by WILLIAM HOOVER.

102. Proposed by CHARLES C. CROSS, Whaleyville, Va.

Required the least multiple of 17 which when divided by 2, 3, 4, . . . 16 leaves in each case 1 as a remainder.

I. Solution by MARVIN E. SMITH, A. M., Instructor in Mathematics, Randolph-Macon Academy, Bedford City, Va.

$x(2^4\cdot 3^2\cdot 5\cdot 7\cdot 11\cdot 13)+1$, x being any positive integer, represents all numbers satisfying the given condition. In order to determine what value of x will make this number the least multiple of 17, the following equation must be solved in least positive integers.